Conversion of units - How to

By Dr. Hulan E. Jack Jr.

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1 Introduction

Every term in an equation must not only have the same physical dimensions, but also the same units. So converting from one unit to another is a very important task. And one that you will frequently encounter. We will develop a formal method for units that are strictly proportional to each other by way of an example. This method does not work for conversions that have an additive constant like converting temperature between Fahrenheit(the U.S.) and Centigrade (the rest of the world). They are better handled graphically. There are two major advantages of the method developed here.

a. It is programmatic. That is before using any numbers you lay out a program to get from the units you have to the units you want. Only after completing the program do you use the numbers.

b. It is "self correcting" in the sense that errors are made obvious and easy to correct. The steps are as follows.

It has three steps.

1. Step One consists of knowing where we are and where we want to get to, and deciding on how to make the trip. This step builds the program. It has no numbers, just units.

2. Step Two consists of putting in the appropriate conversion numbers.

3. Step Three is checking the conversion units.

As we will see from the examples, this method has a number of advantages over the method of using a series of individual proportions. First, comes the program. Step One lays out the individual detailed steps that you are going to use in making the conversion. As we will see, the program depends on the specific conversions that you plan to use. Step Two allows for the checking in Step Three. Errors can be found and corrected on inspection. The only way to check the method of using a series of individual proportions is to redo each of the individual

$$mi(5280)\frac{f}{5280=mi}(12)\frac{in}{12in=f}(2.54)\frac{cm}{2.54cm=in}(\frac{1}{100})\frac{m=100}{cm}(\frac{1}{1000})\frac{km=1000}{m} \quad (3)$$

proportions in detail while checking for both unit layout and the numbers - even if you do not rewrite anything, you still must redo each step in detail. If you have made an error along the way you may not find it on the first checking, even on checking it many times in that sitting. You may only catch it at some later distant time! Maybe!!!

2. Defining Example

Let's suppose that we have 5 miles (mi) and we want to know how many kilometers (km). Let's suppose that we know 1mi = 5280 f, 1 f = 12 inch, 1 inch = 2.54 cm, 1m = 100 cm and 1 km = 1000m.. We are going to use these for the conversion.

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Step One: Layout the program.

The scheme is to set up a series of canceling steps that take us from the original unit (what we have) to the final unit (what we want) through a series of intermediate units. In this example we have mi and want to get km. From the units conversions values given above we will make the following trip:

First, you might want to set up a high level **pseudo-code** shown below. The information to be used requires the following sequence of conversions.

mi->f, f->inch, inch-> cm, cm -> m, and finally m -> km.

This is a sequence that lays out the steps we want to execute without the details of how to do it. But, this looks like a sequence of separate steps. We are actually going to do in one single step as

mi -> f -> inch -> cm -> m

The program takes advantage of the fact that for the operations of multiplication and division, units behave like natural numbers (1,2,3,4,.... positive integers without 0). Hence, they cancel. This allows us to setup the sequence of successive cancellations, shown below, to accomplish the objectives laid out in the pseudo-code above.

$$mi(\quad)\frac{f}{mi}(\quad)\frac{in}{f}(\quad)\frac{cm}{in}(\quad)\frac{m}{cm}(\quad)\frac{km}{m} \qquad (1)$$

Notice the pattern of cancellation of units - we start with mi, and by successive cancellation, end with km. We know the program is correct if it is a sequence of cancellations that get you from the unit you have to the unit you want using the known intermediate units.

Step Two: Now enter the numbers for each of the conversions.

$$mi(5280)\frac{f}{mi}(12)\frac{in}{f}(2.54)\frac{cm}{in}(\frac{1}{100})\frac{m}{cm}(\frac{1}{1000})\frac{km}{m} \qquad (2)$$

Step Three: check for the correctness of the numerical entries.

For example, substituting (5280f=mi) , the first term cancels to 1.

$$mi(5280)\frac{f}{5280=mi}(12)\frac{in}{12in=f}(2.54)\frac{cm}{2.54cm=in}(\frac{1}{100})\frac{m=100}{cm}(\frac{1}{1000})\frac{km=1000}{m} \qquad (3)$$

Suppose we mistakenly had written

$$(100)\frac{m}{cm}, \quad doing\ the\ checking\ gives\ (100)\frac{m(=100)}{cm} \qquad (4)$$

This does not cancel. The checking tells us that the number is upside down. So correct it.

Finishing it gives

$$5mi = 5\times(\frac{5280\times12\times2.54}{100\times1000})km = 5\times1.609\,km = 8.047\,km \quad (5)$$

3. Another Example - Choices

The program used for a conversion is not fixed, it depends on what information you are using. Let's look at the same example, but with a different set of intermediate conversions. Let's again do **5mi = ?km** using the conversions

1f = 12 in, 1m = 39.37in and 1km = 1000 m.

Notice that go directly from in -> m without the in->cm step used above. The new solution follows.

<u>Step One</u>: We layout **the program**.

Now the pseudo-code is

mi -> f, f -> in in -> m, m ->k m

and the appropriate program becomes

$$mi(\quad)\frac{f}{mi}(\quad)\frac{in}{f}(\quad)\frac{m}{in}(\quad)\frac{km}{m} \quad (6)$$

Note that two piece is different from the first example - the in ->cm and cm -> m. do not appear because we do not do the in->cm step .

<u>Step Two:</u> Now enter **the numbers** for each of the conversions.

$$mi(5280)\frac{f}{mi}(12)\frac{in}{f}(\frac{1}{39.37})\frac{m}{in}(\frac{1}{1000})\frac{km}{m} \quad (7)$$

You do the rest.

4. Some more examples

4.1. mph to in/s

Step One: Layout the Program

Find 60 mph = ? f/s using 1 mi = 5280 f, 1 f = 12 in, 1 hr = 60 min, 1 min = 60 s.

Pseudo-code 1) mi -> f, f -> in; 2) Hr -> min, min -> s.

$$60mph = 60\frac{mi(\,)\frac{f}{mi}(\,)\frac{in}{f}}{hr(\,)\frac{min}{hr}(\,)\frac{s}{min}}$$

Step Two: Fill in the numbers

$$60mph = 60\frac{mi(5280)\frac{f}{mi}(12)\frac{in}{f}}{hr(60)\frac{min}{hr}(60)\frac{s}{min}}$$

You finish it.

4.2. The 4 minute mile

"The four minute mile, in athletics, is the running of a mile (1609 metres) in under four minutes. It was once thought to be impossible but has now been achieved by many male athletes..." "On May 6, 1954, the Englishman Roger Bannister ran the first sub-four-minute mile in recorded history at 3 minutes 59.4 seconds." http://en.wikipedia.org/wiki/Four_minute_mile

When running a mile in 4 minutes, how fast is a person traveling in miles per hour, mph?, feet per second, f/s ? Meters per second, m/s? Kilometer per hour, km/hr?

We suppose we know 1 mi = 5280 f, 1 km = 0.62 mi, 1 min = 60 s (sec), and 1 hr = 60 min.

a. For 1mile/4min = ? F/s requires two conversion; mi -> f on top (the numerator) and min -> s on the bottom (denominator).

The pseudo code is numerator mi -> f ; denominator min -> s .

$$\frac{1mi}{4\min} = \frac{1mi()\frac{f}{mi}}{4\min(\ )\frac{s}{\min}} = \frac{1mi(5280)\frac{f}{mi}}{4\min(60\ )\frac{s}{\min}} = \frac{1(5280)f}{4\ (60\ )s} = \frac{1\ (1320)\ f}{(60\ )s} = 22\ f/s$$

b. For 1mil2/4min = ? km/hr requires two conversion; mi -> km on top (the numerator) and min -> hr on the bottom (denominator).

The pseudo code is numerator mi -> km ; denominator min ->hr .

$$\frac{1mi}{4\min} = \frac{1mi()\frac{km}{mi}}{4\min(\ )\frac{hr}{\min}} = \frac{1mi(\frac{1}{0.62})\frac{km}{mi}}{4\min(\frac{1}{60})\frac{hr}{\min}} = \frac{1(\frac{1}{0.62})km}{4\ (\frac{1}{60}\ )hr} = \frac{1\ (\frac{1}{0.62})\ km}{(\frac{1}{15}\ )hr} = \frac{1\ (15)\ km}{(0.62\ )hr} = 24.2\,km/hr$$

You show that it is 15 mph.

4.3. Newton (N) -> Pound (p) Conversion

To convert between Newton (the International [metric] standard for force) and the pound (the English - U.S. unit for force) requires Newton's 2nd Law of Motion, F = ma. The reason for this is as follows. The International System of Measures uses the fundamental dimension mass

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in the unit of kilogram as its standard. In this system force is a derived dimension and its unit is the Newton. The English system uses force as its fundamental dimension by way of weight in the unit of pound (p). The relation between the two units , kg and p must be directly measured. On earth where g = 9.81 m/s^2 and 1kg weighs 2.206 p. (Note that g varies over the earth. Values between 980 and 981 are typically used.) This result of the measurement depends on location. For instance, on the moon the result is about 1/6th this value because the value of g on the moon is about 1/6th that on earth. So, we must travel the path laid out in this annotated pseudo-code

N (force) -> kg (m = F/g = W /g) -> p (force) .

The program with conversions numbers is

$$1N = 1N(\ \ )\frac{kg}{N}(\ \ )\frac{p}{kg} = 1N(\frac{1}{9.81})\frac{kg}{N}(2.206)\frac{p}{kg} = 0.2249\,p$$

So, 1 N = 0.2249 p and its reciprocal gives 1 p = 4.447 N. This conversion is true everywhere because the measured value of (p /kg) is proportional to the value of g. For example, g on the moon is about 1/6th that on earth, so measured value of p/kg is equally reduced.

For instance on earth 1kg x 9.81 m/s^2 = 9.81 N. Thus 1 kg weighs 9.81 N on earth. But, on the moon 1 kg weighs proportionately less, about 1/6.

4.4 Joules -> fp (foot pound) Conversion.

Here we use the initial data kg = 9.81N, 1 kg = 2.206p on earth, 1m = 100cm, 1in = 2.54 cm, and July 26, 20001 f = 12 in

$$1J = 1Nm = 1N \times m = 1N(\frac{1}{9.81})\frac{kg}{N}(2.206)\frac{p}{kg} \times m(100)\frac{cm}{m}(\frac{1}{2.54})\frac{in}{cm}(\frac{1}{12})\frac{f}{in} = 0.7378\,fp$$

5. Exercises:

1. Show that 3 mph (miles per hour = mi/hr) = 4.40 f/s, = 1.34 m/s, = 4.82 km/h.
   Use 1mi = 5,280f, 1f = 12in, 1in = 2.54 cm, 1m = 100cm, 1km = 1,000m.
2. Show that 1 f^2 (1 square foot) = 144 i^2 , = 929.0 cm^2 = 0.0929 m^2
   Use the units listed above.